Culture-area relation in Axelrod's model for culture dissemination

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Axelrod's model for culture dissemination offers a nontrivial answer to the question of why there is cultural diversity given that people's beliefs have a tendency to become more similar to each other's as people interact repeatedly. The answer depends on the two control parameters of the model, namely, the number F of cultural features that characterize each agent, and the number q of traits that each feature can take on, as well as on the size A of the territory or, equivalently, on the number of interacting agents. Here we investigate the dependence of the number C of distinct coexisting cultures on the area A in Axelrod's model – the culture-area relationship – through extensive Monte Carlo simulations. We find a non-monotonous culture-area relation, for which the number of cultures decreases when the area grows beyond a certain size, provided that q is smaller than a threshold value $q_c = q_c(F)$ and $F \geq 3$. In the limit of infinite area, this threshold value signals the onset of a discontinuous transition between a globalized regime marked by a uniform culture ($C = 1$), and a completely polarized regime where all $C = q^F$ possible cultures coexist. Otherwise the culture-area relation exhibits the typical behavior of the species-area relation, i.e., a monotonically increasing curve the slope of which is steep at first and steadily levels off at some maximum diversity value.

I. INTRODUCTION

Axelrod's model for the dissemination of culture or social influence [2] is a paradigm for idealized models of collective behavior which seek to boil down a collective phenomenon to its functional essence [7]. The main issue the model addresses is why cultural differences persist despite the fact that interactions between people tend to make them more alike in their beliefs and attitudes. Building on just a few simple principles, Axelrod's model provides a highly nontrivial answer to that question. In that model, an agent is represented by a string of cultural features, where each feature can adopt a certain number of distinct traits. The interaction between any two agents takes place with probability proportional to their cultural similarity, i.e., proportional to the number of traits they have in common. The result of such interaction is the increase of the similarity between the two agents, as one of them modifies a previously distinct trait to match that of its partner.

A remarkable aspect of Axelrod's model is that, notwithstanding the built-in assumption that social actors have a tendency to become similar to each other through local interactions, the model exhibits global polarization, i.e., a stable multicultural regime [2]. Subsequent analysis of this model by the statistical physics community has revealed a rich dynamic behavior with a nonequilibrium phase transition separating the global polarization regime from the homogeneous regime, where a single culture dominates the entire population [3, 13, 14, 26]. An important outcome of those more quantitative studies was the finding that the multicultural regime

is unstable to a vanishingly small noise that allows for the agents to spontaneously change their opinions [15] (see, however, [22]). Several studies of a more qualitative character have considered generalizations of the original model such as variability in the agents' range of communication and mass media effects (see, e.g., [8, 12, 29]). These efforts seem to have established Axelrod's model as the reference minimal model of social influence or culture dissemination both in the social and physical sciences [26, 34]. We must note, however, that there are many alternative models of social influence or opinion formation which, similarly to Axelrod's, focus on the interplay between consensus and diversity, and which have also been extensively studied by the statistical physics community (see, e.g., [4, 6, 18, 33]).

Despite all the interest raised by Axelrod's model, a most appealing outcome of the model – the existence of a multicultural regime – has been somewhat overlooked and even obvious questions such as the relation between the number of coexisting cultures and the area available to the social agents, i.e., the culture-area relation has not been fully addressed. This is surprising in view of the counterintuitive result found by [2] that the number of coexisting cultures decreases when the area grows beyond a certain size, which starkly contrasts with the biological species-area relations characterized by the monotonical increase of the number of species with the size of the area of a particular habitat [9, 24, 25, 32].

Axelrod's model is characterized by two integer-valued parameters, namely, the list of features or dimensions of culture F and the number of traits q which are the possible values each feature can take on. The social agents live in the sites of a square lattice of linear size L and can interact with their nearest neighbors only. In this contribution we re-examine the culture-area relation in Axelrod's model and show that the unusual non-monotonic behavior occurs only in the regime of $F \geq 3$ and $q < q_c$

where $q_c = q_c(F)$ is the number of traits at which the discontinuous transition takes place in the limit $L \to \infty$. Otherwise, the culture-area relation exhibits the typical behavior of the species-area relation – a monotonically increasing curve the slope of which is steep at first and steadily levels off at the maximum diversity value q^F. Since there is no 'intuitive' reason for the discontinuous transition to take place at a particular value of q (or to not occur for $F = 2$, for instance), we think the non-monotonical behavior of the culture-area relation has no first-principle explanation, as sought by [2] in his original work.

This study is organized as follows: In Sect. II we describe briefly Axelrod's model for culture dissemination and in Sect. III we present and discuss the culture-area relations obtained from extensive simulations of the two representative cases $F = 2$ and $F = 3$ for which the transition between the multicultural and homogeneous regimes is continuous and discontinuous, respectively. Finally, in Sect. IV we relate our findings to results of models of language competition [28, 31] and discuss a possible connection between Axelrod's model and the Derrida-Higgs model for sympatric speciation [10, 11, 19].

II. MODEL

As pointed out before, in Axelrod's model each agent is characterized by a set of F cultural features which can take on q distinct values. The agents are fixed in the sites of a square lattice with open boundary conditions (i.e., agents in the corners interact with two neighbors, agents in the sides with three, and agents in the bulk with four nearest neighbors). The social agents can be thought of as individuals or as homogeneous villages. The initial configuration is completely random with the features of each agent given by random integers drawn uniformly between 1 and q. At each time we pick an agent at random (this is the target agent) as well as one of its neighbors. These two agents interact with probability equal to their cultural similarity, defined as the fraction of common cultural features. An interaction consists of selecting at random one of the distinct features, and changing the target agent's trait on this feature to the neighbor's corresponding trait. This procedure is repeated until the system is frozen in an absorbing configuration.

Given the bias towards homogenization, it is really remarkable that in some cases the system can reach a multicultural absorbing state. We recall that the sources of disorder in Axelrod's model are the stochastic update sequence and the choice of the initial configuration: it is the competition between the disorder of the initial configuration and the ordering bias of the local interactions that is responsible for the nontrivial threshold phenomenon reported by [3].

III. SIMULATIONS

A feature that sets our results apart from those reported previously in the literature is that our data points represent averages over at least 10^3 independent runs. This requires a substantial computational effort, especially in the regime where the number of cultures decreases with the lattice size since then the time for absorption can be as large as $10^6 \times A$ where $A = L^2$ is the lattice area. In the figures presented in the following, the error bars are smaller or at most equal to the symbol sizes.

To simulate efficiently Axelrod's model we make a list of the active agents. An active agent is an agent that has at least one feature in common and at least one feature distinct with at least one of its nearest neighbors. Clearly, since only active agents can change their cultural features, it is more efficient to select the target agent randomly from the list of active agents rather than from the entire lattice. Note that the randomly selected neighbor of the target agent may not necessarily be an active agent itself. In the case that the cultural features of the target agent are modified by the interaction with its neighbor, we need to re-examine the active/inactive status of the target agent as well as of all its neighbors so as to update the list of active agents. The dynamics is frozen when the list of active agents is empty.

Our focus is on the number of distinct cultures C, rather than on the number of clusters or the fraction of the lattice occupied by the largest cluster [3, 13, 14]. In that sense, cultural diasporas, which occurs when regions with specific cultural features are disconnected from other regions with the same cultural features [8], are counted as a single culture. Of course, C is much easier to compute than the cultural regions and, as we will show next, provides an equally good indication of the existence and location of a threshold phenomenon. In addition, we consider the cases $F = 2$ and $F = 3$ only as, according to [3], the dynamic and static properties of Axelrod's model for $F > 3$ are qualitatively similar to those for $F = 3$.

Figure 1 exhibits the culture-area relation for $F = 3$ and different values of q. The non-monotonic behavior reported by [2] appears for $q < q_c = 16$ only. For $q \geq q_c$ the number of cultures increases linearly with increasing A at first and then gradually flattens when A becomes on the order of the maximum value, q^F. In the limit $L \to \infty$ we have only two possible outcomes: if $q < q_c$ then $C \to 1$ and a single culture dominates the lattice (ordered regime), otherwise $C \to q^F$ and all cultures are represented in the lattice (disordered regime). The transition between these two regimes is discontinuous because C jumps from 1 to q^F at $q = q_c$. As mentioned before, this behavior is expected to occur for all $F \geq 3$ with the threshold value $q_c = q_c(F)$ increasing monotonically with increasing F [3].

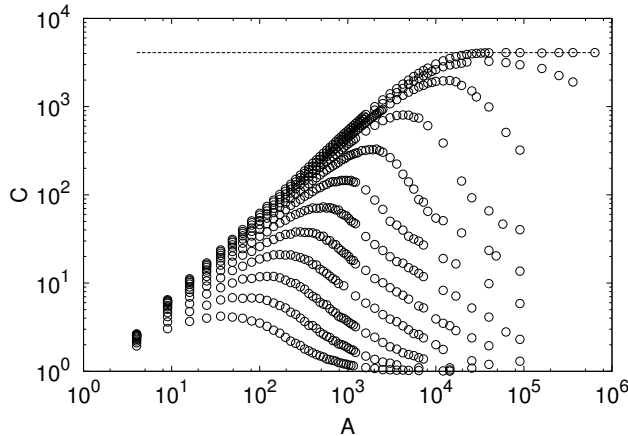Figure 2 summarizes our findings regarding the case $F = 2$. The first point to be noted is that, in contrast

FIG. 1: Logarithmic plot of the culture-area relation for $F = 3$ and (bottom to top) $q = 5, 6, \ldots, 16$. The dashed horizontal line indicates the value $C = 16^3$. In the limit $A \to \infty$ there are two distinct regimes: either $C \to 1$ ($q < 16$) or $C \to q^F$ ($q \geq 16$).
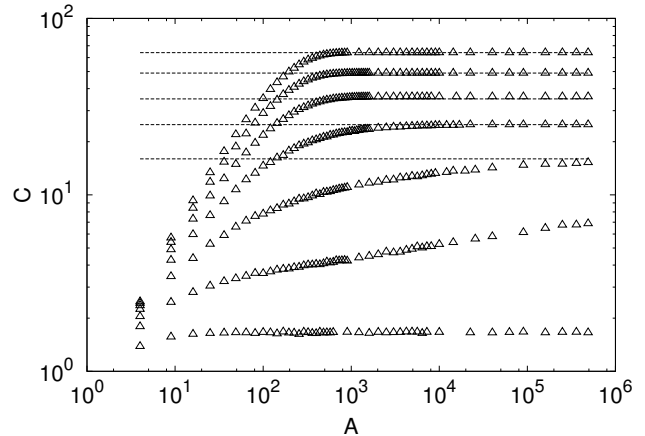

FIG. 2: Logarithmic plot of the culture-area relation for $F = 2$ and (bottom to top) $q = 2, 3, \ldots, 8$. The dashed horizontal lines indicate the maximum diversity values q^F to which the data for $q \geq 4$ converge in the limit of infinite area.

with the previous case, the culture-area relation exhibits the expected monotonic behavior, which implies that the globally homogeneous regime $C = 1$ does not appear in the limit $A \to \infty$. But the disordered regime, which is characterized by the coexistence of all q^F cultures, is present as revealed by the data for $q \geq 4$. We can identify a second regime (see data for $q = 2$), in which only a fraction of the total number of cultures coexist in the limit of infinite lattices. In this limit we find $C \to 1.66 \pm 0.01$ for $q = 2$. It is not clear whether the data for $q = 3$ will ultimately converge to $C = 9$: for $L = 700$ we find $C = 6.91 \pm 0.15$ but the data show a trend to increase much further. The very slow convergence may indicate that $q = 3$ is the threshold (critical) value that separates the two regimes. In the case q is allowed to change continuously (for example, by choosing the trait values as samples of a Poisson distribution of mean q), [3] have shown that the transition between these two regimes is continuous, in the sense that, for $A \to \infty$, C increases continuously from 1 to q_c^2 as the mean of the Poisson distribution varies from 0 to q_c.

It is instructive to calculate the number of cultures in the totally disordered initial configuration, in which the $A = L^2$ agents are assigned one of the any q^F cultures. This is a classical occupancy problem discussed at length in Feller's book [5, Ch. IV.2]. In this occupancy problem, the probability that exactly m cultures are not used in the assignment of the A agents to the q^F cultures is

$$P_m \left(A, q^F \right) = \binom{q^F}{m} \sum_{\nu=0}^{q^F - m} \binom{q^F - m}{\nu} (-1)^\nu \left(1 - \frac{m + \nu}{q^F} \right) \tag{1}$$

which in the limit where A and q^F are large reduces to the Poisson distribution

$$p \left(m; \lambda \right) = \mathrm{e}^{-\lambda} \frac{\lambda^m}{m!} \tag{2}$$

where $\lambda = q^F \exp \left(-A/q^F \right)$ remains bounded [5, Ch. IV.2]. Hence the average cultural diversity C_r resulting from the random assignment of agents to cultures is simply $q^F - \langle m \rangle$, which yields

$$C_r = q^F \left[1 - \exp \left(-A/q^F \right) \right] . \tag{3}$$

This quantity is a monotonically increasing function of A which grows linearly in the regime $A \ll q^F$ and tends to the maximum diversity value q^F when $A \gg q^F$. Figure 3 shows a comparison between the predictions of Eq. (3) and the simulation data of Axelrod's model in the global polarization (multicultural) regime. Although the random occupancy hypothesis yields a good qualitative description of the culture-area relations in this regime, it consistently overestimates the values for the cultural diversity. This is expected as the effect of the local interactions in Axelrod's model is to decrease the cultural differences between neighboring agents.

IV. CONCLUSION

In contrast with the species-area relation of Biology for which there are plenty of field data to check the theoretical proposals [24], in the culture-area relation there are practically no empirical evidences to back any quantitative theoretical prediction. However, there are some empirical results regarding the language-area relation [20, 21], which are appropriate to mention here since the mechanisms of development, dissemination and acquisition of language are similar, if not identical, to those of culture. An extensive analysis of the language diversity that considers ecological and linguistic variables for about 74 countries yields $C \propto A^x$ with $x = 0.5 \pm 0.1$ [20]. In the region $q^F \gg A$, Axelrod's model yields also a power-law scaling but with the exponent $x = 1$. Given the crudeness of the model and the inherent difficulties
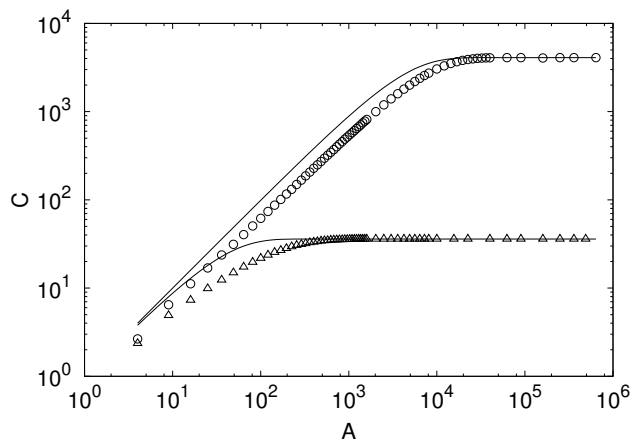
FIG. 3: Comparison between the random initial diversity C_r (solid lines) given by Eq. (3) and the stationary diversity of Axelrod's model for $F = 3$, $q = 16$ ($\circ$), and $F = 2$, $q = 6$ ($\triangle$).

involved in counting languages, either agreement or disagreement on this matter seems to be of little significance. Nevertheless, it should be interesting to find out whether changes in rules for the local interaction between agents can affect the value of that exponent. We note that the area A used in the field studies is the area of the country, whereas in Axelrod's model A is the area of the lattice or, equivalently, the number of agents (population size). The exponent x is not affected by these different interpretations of A, provided the population size grows linearly with the territory area.

Interestingly, extensive Monte Carlo simulations of a language competition model [28, 31] yield a non-monotonic relation between the number of languages and the number of speakers (agents). In fact, the similarity between the language-area (population size) relation obtained in the case the individuals are placed in the sites of a scale-free network (see Fig. 4 of [28]) and the relations shown in Fig. 1 for $q < 16$ is striking. The model for language competition proposed by [28] has an important element in common with Axelrod's model – a language is defined by F independent features each of which can take one of q different values. The local interaction rules, however, are completely distinct and, for instance, the similarity between the agents' languages has no role in determining the occurrence of an interaction. Unless there is an explicit dependence of the usefulness of a language on spatial coordinates [23], the ultimate outcome of the language competition models is the dominance of a single language [1] (see, however, [28] for an overview of language models which may exhibit language coexistence) and so since the source of diversity or randomness is identical in both models (the initial distribution of languages and cultures) the similarities pointed out may not be so surprising after all.

A word is in order about the relevance of Axelrod's model for theoretical biology and, in particular, for the description of the dynamics of species in a habitat. From the broad perspective, we must point out that the case for the pertinence of culture to the understanding of the hominid evolutionary process has been persuasively advocated by [17] using the biological concept of niche construction [16]. More parochially, however, we can interpret Axelrod's model as a model for sympatric speciation based on a mate choice mechanism that depends on the similarity (genetic distance) between mates (see [27] for a review of several mechanisms by which sexual selection can drive speciation). One such model is the Derrida-Higgs model of species formation [10, 11, 19] in which mating only occurs between individuals that are genetically similar to each other. This assumption is akin to the restriction of the interactions in Axelrod's model to individuals that share a certain number of cultural traits. In addition, we can re-interpret cultural assimilation by the target agent as the result of a sexual reproduction scheme implemented by a Moran-type stochastic process in which the target agent is replaced by the offspring whose genotype is identical to the target's genotype except for a single gene which is inherited from the other mate. Although a more traditional crossover scheme is unlikely to change qualitatively the outcome of the competition between dominance and diversity discussed in this paper, it would be interesting to study the Derrida-Higgs model of species formation with the individuals fixed in the lattice sites and the mating restricted to their nearest neighbors as in Axelrod's model.

The paucity of empirical data to support and motivate the proposal of models for culture dissemination and social influence is about to change as more people become connected by the Web 2.0 social networks. The on-line communities in these networks can provide an invaluable source of data to validate theoretical predictions of models such as Axelrod's. In fact, the basic idea that agents who are similar to each other are more likely to interact ('birds of a feather flock together') and then become even more similar was observed in that context by [30]. Analysis of a population of over 10^7 people indicates that people who chat with each other using instant messaging are more likely to have common interests, as measured by the similarity of their Web searches, and the more time they spend talking, the stronger this relationship is.

Acknowledgments

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